ABSTRACT

This paper discusses the tracking control problem for a class of multi-input-multi-output (MIMO) mismatched linear systems, where there are disturbances in different channels from the control input and the real-time feedback signal is not the output of interest. This mismatch makes it difficult to achieve high tracking performance for the interested output. To address this problem, two model based iterative learning control (ILC) algorithms, namely reference ILC and torque ILC, are designed for different injection locations in the closed loop system. An ad hoc hybrid scheme is proposed to make transitions between the two ILC stages for them to work properly at the same time. The proposed scheme is validated through the experimental study on a single-joint indirect drive system.

INTRODUCTION

In industrial applications, the automated system (e.g., the robotic manipulator) is often required to repeatedly perform a single task under the same operating conditions. If the system repeatability is good, the trajectory tracking error will become repetitive from one run to another. In this case, the iterative learning control (ILC) scheme is well suited to compensate for the repeatable tracking error [1, 2].

Iterative learning control is a data-driven methodology which iteratively utilizes the data (e.g., error profile) from previous trails to update the system inputs for the next iteration. Many variations of the ILC scheme have been studied for various applications [1]. Most of them, however, are developed for the fundamental case where the system has direct measurement of the interested output for real-time feedback and does not have disturbances in the channels different from the control input. Therefore, in mismatched systems where the above scenario does not hold, the standard ILC performance will be limited and in some cases the ILC convergence is hard to guarantee.

The mismatched systems discussed in this paper are common in practical applications, e.g., the industrial robots with indirect-drive joint mechanisms (joints with elasticity). Several feedback control approaches, such as integrator backstepping [3], dynamic surface control [4], and adaptive robust control [5], have been developed specifically to deal with these mismatched systems. Some efforts have been devoted to migrating these ideas to the field of ILC to deal with the mismatched uncertainty iteratively while exploiting the noncausal repetitiveness. A two-stage ILC approach was proposed in [6] to deal with robots with joint elasticity. Similar to backstepping, the real-time measured output (i.e., motor side state) is utilized in [6] as a hypothetical input to control the output of interest (i.e., load side state). As shown later in this paper, the convergence rate of this learning process may be adversely affected and thus the use of a high bandwidth \( Q \) filter to learn high frequency error may compromise stability. Other studies such as [7] also reported the compromise on the tracking performance they had to make for a better learning convergence. This is especially the case when the system exhibits mismatched uncertainties. Regarding this stabil-
ity issue under uncertainty, various robust approaches have been proposed [8, 9]. The resulting algorithms are usually nontrivial and the performance is normally compromised for a conservative robust controller. In [9], it requires the plant resonances to be suppressed by feedback compensation in order for the proposed method to improve the robustness to high-frequency modeling errors. In [10], a model-based ILC approach was developed for elastic robots to learn the error component beyond the first resonant frequency. However, this approach requires an accurate piecewise-linear model to be identified and interpolated for each trajectory in advance, which limits its application.

This paper will propose a hybrid two-stage model-based ILC approach for a class of MIMO mismatched linear systems. The two-stage ILC is aimed to push the learning algorithm to a higher bandwidth while maintaining the fast model-based convergence rate. The paper is organized as follows. The system model and the basic controller structure are introduced first. Then two ILC schemes are designed independently followed by an ad hoc hybrid scheme to enable the two ILC schemes to execute simultaneously. The experimental study on a single-joint indirect drive system is presented next to validate the effectiveness of the proposed scheme. The parametric uncertainty and mismatched dynamics such as various disturbances at different locations of the system will be addressed. The conclusions of this work will be given at last.

SYSTEM OVERVIEW
System Model
Consider a MIMO mismatched linear system in the following discrete time state space form

\[ x(j+1) = Ax(j) + Bu(j) + Bd(j) \]
\[ y(j) = [q_m(j), q_e(j)]^T = Cx(j) + Du(j) + D_d d(j) \]

where \( j \) is the time step index, \( x \in \mathbb{R}^{n_x} \) is the system state, \( u \in \mathbb{R}^{n_u} \) is the control input, \( d \in \mathbb{R}^{n_d} \) is the lumped disturbance, \( q_m \in \mathbb{R}^{n_m} \) and \( q_e \in \mathbb{R}^{n_e} \) are the two outputs of the plant. \( d \) is regarded as the mismatched uncertainty/disturbance if it (or part of it) is applied to different channels from the control input \( u \) (i.e., \( B_d \neq \alpha B_d, \forall \alpha \in \mathbb{R} \)). Another mismatched assumption is that, only part of the outputs (i.e., \( q_m \)) is measured for real-time feedback, even if the output of interest may be \( q_e \). However, the measurement of \( q_e \) may be available for iteration based off-line use. Furthermore, besides the unknown mismatched dynamics, it is assumed that parametric uncertainties exist in the available nominal model.

This system can be reformulated as

\[ q_m(j) = P_{mu}(z)u(j) + P_{md}(z)d(j) \]
\[ q_e(j) = P_{ea}(z)u(j) + P_{ed}(z)d(j) \]

Basic Controller Structure
Figure 1 illustrates the control structure for this mismatched system, where the subscript \( k \) is the iteration index. It consists of two feedforward controllers, \( F_1 \) and \( F_2 \), and one feedback controller, \( C \). Here, \( C \) can be any linear feedback controller such as a decoupled PID controller to stabilize the system. The feedforward controllers, \( F_1 \) and \( F_2 \), are designed using the nominal inverse model as

\[ q_{md,k}(j) = \hat{P}_{mu}(z)\hat{P}_{td}^{-1}(z)q_{ld,k}(j) \Rightarrow F_1(z)q_{ld,k}(j) \]
\[ q_{ln,k}(j) = \hat{P}_{mu}^{-1}(z)q_{md,k}(j) + r_{q,k}(j) \Rightarrow F_2(z)q_{md,k}(j) \]

where \( \bullet \) is the nominal model representation of \( \bullet \), \( q_{ld,k} \) is the desired plant output for \( q_{l,k} \), \( r_{q,k} \) and \( q_{md,k} \) are used as the additional reference and feedforward torque updates generated iteratively by the two-stage ILC algorithm designed later.

TWO-STAGE ILC SCHEME
ILC Basics
Some basics of the general ILC scheme are reviewed first, which will be utilized in the subsequent ILC scheme design. Consider the MIMO linear system with the error dynamics and the ILC law in the following form

\[ \tilde{e}_1(j) = -P_{mu}(z)\tilde{u}_1(j) + \tilde{r}(j) \]
\[ \tilde{u}_{k+1}(j) = Q(z) [\tilde{u}_k(j) + L(z)\tilde{e}_k(j)] \]

where \( \tilde{e} \) is the error that the ILC scheme aims to reduce, \( \tilde{r} \) is the lumped repetitive reference and/or disturbance input to the
system, and \(\bar{u}\) is the control input updated iteratively by the ILC scheme using the filters \(L(z)\) and \(Q(z)\). Similar to [1, 11], the following convergence property holds:

**Theorem 1.** The ILC system (7)-(8) is monotonically and exponentially convergent in the sense that \(\|\bar{u}_k - \bar{u}_\infty\|_p \to 0\) and \(\|\bar{e}_k - \bar{e}_\infty\|_p \to 0\) as \(k \to \infty\), if

\[
\beta = \|Q(z) [I - L(z)P_{eu}(z)]\|_p < 1
\]

where \(\beta\) is the rate of convergence, \(I\) is the identity matrix with appropriate dimension, the \(p\)-norm \(\|\cdot\|_p = (\sum_i |\cdot|^p)^{1/p}\), and

\[
\bar{u}_\infty(j) = [I - Q(z) + Q(z)L(z)P_{eu}(z)]^{-1} Q(z)L(z)\bar{r}(j) \quad (10)
\]

\[
\bar{e}_\infty(j) = [I - Q(z) + Q(z)L(z)P_{eu}(z)]^{-1} [I - Q(z)] \bar{r}(j) \quad (11)
\]

**Proof.** First, with (7) and (8), it is easy to see that

\[
\bar{u}_{k+1}(j) = Q(z) [I - L(z)P_{eu}(z)] \bar{u}_k(j) + Q(z)L(z)\bar{r}(j) \quad (12)
\]

which yields

\[
\|\bar{u}_{k+1} - \bar{u}_\infty\|_p = \|Q(z) [I - L(z)P_{eu}(z)] (\bar{u}_k - \bar{u}_\infty)\|_p \\
\leq \|Q(z) [I - L(z)P_{eu}(z)]\|_p \|\bar{u}_k - \bar{u}_\infty\|_p \quad (13)
\]

Therefore, if \(\|Q(z) [I - L(z)P_{eu}(z)]\|_p < 1\), \(\|\bar{u}_k - \bar{u}_\infty\|_p \to 0\) as \(k \to \infty\). With (7), similar conclusion can be drawn for the convergence of \(\bar{e}_k\). Note that the inverse \([I - Q(z) + Q(z)L(z)P_{eu}(z)]^{-1}\) exists because \(\|Q(z) [I - L(z)P_{eu}(z)]\|_p < 1\). \(\Box\)

Equation (11) shows that the steady state error \(\bar{e}_\infty\) vanishes with complete learning (i.e., \(Q(z) = I\)), which means the effects on \(\bar{e}_\infty\) from the repetitive input \(\bar{r}\) will be fully compensated. The ILC law (8) implies the \(Q\) filter can also be used to shape the learning ability in the frequency domain. In order to achieve better performance, it is desired to push the bandwidth of \(Q(z)\) to be as high as possible. Equation (9), however, indicates that the bandwidth of \(Q(z)\) may have to be compromised to ensure monotonic convergence and to avoid poor transients in the learning process. In practice, a low-pass filter \(Q(z)\) is typically employed to prevent the effects of high frequency model uncertainties from entering the learning process [1]. Also, \(Q(z)\) should be unity gain at low frequencies where complete learning is preferred to achieve zero steady state error. Since the ILC scheme is an off-line iteration based method, acausal filtering can be utilized to obtain a zero-phase learning response.

Given a fixed \(Q\) filter, the optimal learning filter to achieve the fastest convergence is obtained as

\[
L^*(z) = \arg \min_{L(z)} \|Q(z) [I - L(z)P_{eu}(z)]\|_p
\]

This leads to the plant inversion choice, i.e., \(L^*(z) = P_{eu}^{-1}(z)\). This model matching problem can be solved with many optimization techniques, such as the H∞ synthesis [8], if the model uncertainty bound is known. The designed \(Q(z)\) and \(L(z)\) need to be validated using (9) with the knowledge of system model uncertainty. Without loss of generality, the optimal learning filter in this paper is simply chosen as \(L^*(z) = I\).

**ILC With Reference Update**

Denote the sensitivity function of the closed loop system in Fig. 1 as \(S_p(z) = [I + C(z)P_{mu}(z)]^{-1}\). From (5), \(q_{md,k}\) is related to \(q_{d,k}\) (i.e., \(q_{md,k}\) or \(q_{d,k}\)) as follows

\[
q_{md,k} = \hat{P}_{mu} \hat{P}_u^{-1} q_{d,k}
\]

where \(P_s\) can be either \(P_{mu}\) or \(P_{mu}\) depending on the choice of \(q_{d,k}\). The time index \(j\) for all signals and the discrete time operator \(z\) for all transfer functions are omitted hereafter for simplicity. The system output \(q_k\) (i.e., \(q_{md,k}\) or \(q_{d,k}\)) can be derived as

\[
q_k = P_{au}u_k + P_{ud}d_k
\]

\[
= P_{au}S_p \left( C + \hat{P}_u^{-1}(q_{md,k} + r_{q,k}) + \tau_{nl,k} - CP_{mu\mu}d_k \right) + P_{ud}d_k
\]

\[
= \hat{P}_u^{-1} P_{au}S_p \hat{S}_p^{-1} r_{q,k} + P_{au}S_p \left( \hat{P}_u^{-1} \hat{S}_p^{-1} q_{d,k} + \tau_{nl,k} - CP_{mu\mu}d_k \right) + P_{ud}d_k
\]

The corresponding tracking error \(e_k\) is

\[
e_k = q_{d,k} - q_k = -\hat{P}_u^{-1} P_{au}S_p \hat{S}_p^{-1} r_{q,k} + (I - P_{au}S_p \hat{P}_u^{-1} \hat{S}_p^{-1}) q_{d,k} + \tau_{nl,k} - CP_{mu\mu}d_k \]

\[
= -P_{au}S_p \tau_{nl,k} + (P_{au}S_p CP_{mu\mu} - P_{ud})d_k
\]

The tracking performance of the next iteration can be improved with the reference update scheme (namely reference ILC (L) or reference ILC (M) depending on the choice of \(e_k\) as

\[
r_{q,k+1} = \hat{Q}_r r_{q,k} + L_r e_k
\]

Assume the desired trajectory \(q_{d,k}\), the feedforward torque update \(\tau_{nl,k}\), and the disturbance \(d_k\) are repetitive for each iteration. From Theorem 1, the monotonic convergence of this ILC scheme (18) can be guaranteed if \(\beta_r = \|Q_r(1 - L^*_r P_{eu}^{-1} P_{au}S_p \hat{S}_p^{-1})\|_\infty < 1\). With the inversion of the nominal model in (17), the optimal learning filter and the convergence rate are obtained as

\[
L_r^* = \hat{P}_u^{-1} P_{au} S_p \hat{S}_p^{-1} \hat{P}_{mu}^{-1}
\]

\[
\beta_r^* = \|Q_r(1 - \hat{P}_u^{-1} P_{au} S_p \hat{S}_p^{-1})\|_\infty < 1
\]

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and with complete learning (i.e., $Q_r = I$), the tracking error $e_{\infty}$ vanishes. In order to achieve fast convergence rate without compromising the bandwidth of $Q_r$, it is desired to reduce the model uncertainties. This can be done by either obtaining a nominal model $\hat{P}_u$ more accurately representing the actual physical plant, or in contrast, making the inner plant (blue shaded area in Fig. 1) behave as the chosen nominal model $\hat{\mathcal{P}}_u$. In the next section, an ILC scheme with torque update is introduced to achieve the latter objective, i.e., making $q_k \rightarrow \hat{P}_u \mu_k$, where $\mu_k = \tau_{m,k} + \tau_{fb,k}$ is the torque input to the inner plant.

**ILC With Torque Update**

Define $e_{p,k}$ as the model following error between the nominal plant output (i.e., $q_{p,k} = \hat{P}_u \mu_k$) and the actual plant output $q_k$ (i.e., $q_{\ell,k}$ or $q_{m,k}$). The ILC scheme to reduce this model following error $e_{p,k}$ can be formulated as

$$
e_{p,k} = \hat{P}_u \mu_k - q_k \
\tau_{nl,k+1} = Q_u (\tau_{nl,k} + L_a e_{p,k})$$

(21)
(22)

where $\hat{P}_u$ is $\hat{P}_{u_k}$ or $\hat{P}_{m_u}$ to match with the choice of $q_k$. The corresponding ILC is termed as torque ILC (L) or torque ILC (M).

It shows that $\tau_{nl,k}$ is used to cancel out the effects on $q_k$ from model uncertainty $\Delta P \triangleq P_u - \hat{P}_u$ and mismatched disturbance $d_k$. The ideal $\tau_{nl,k}$ to achieve this objective can be derived as

$$\hat{P}_u \mu_k = P_u (\mu_k + \tau_{nl,k}^*) + P_d d_k$$

$$\tau_{nl,k}^* = -P_{u}^{-1}(\Delta P \mu_k + P_d d_k)$$

(23)

In the mismatched systems, the two objectives, following $\hat{P}_{m_u}$ (i.e., torque ILC (M)) and following $\hat{P}_{u_k}$ (i.e., torque ILC (L)), cannot be reached simultaneously (i.e., $\tau_{nl,m} \neq \tau_{nl,l}$), since $P_{m_u}^{-1}P_{m_d} \neq P_{u_l}^{-1}P_{d}$ and $P_{m_u}^{-1} \neq P_{u_l}^{-1}P_{d}$ in general. This means at this stage it is desired to select the nominal model to match with the chosen one in the reference ILC stage.

**Convergence of Model Following Error** The input-output equation of the nominal plant can be derived as

$$q_{p,k} \triangleq \hat{P}_u \mu_k = \hat{P}_u S_p [(C + \hat{P}_{m_u}^{-1})(q_{m,k} + r_{q,k}) - C P_{m_u} \tau_{nl,k} - C P_{m_d} d_k]$$

(24)

Then from (16) and (24), the model following error $e_{p,k}$ becomes

$$e_{p,k} = q_{p,k} - q_k = -T_u S_p \tau_{nl,k} - \Delta P S_p (C + \hat{P}_{m_u}^{-1})(q_{m,k} + r_{q,k}) + (\Delta P S_p C P_{m_d} - P_d) d_k$$

$$\triangleq -P_{cu,u} \tau_{nl,k} + \bar{r}_{u,k}$$

(25)

where $T_u = \bar{P}_u C P_{m_u} + P_a$.

Therefore, if the desired trajectory $q_{m,k}$, the reference update $r_{q,k}$, and the disturbance $d_k$ remain the same for each iteration, by Theorem 1, the torque ILC scheme (22) will be monotonically converging if $\beta_u = \|Q_u (I - L_a T_u S_p)\|_\infty < 1$. By using the nominal plant inversion, the optimal choice of $L_u$ with deadbeat convergence rate $\beta_u^*$ becomes

$$L_u^* = \hat{P}_{u,u}^{-1} S_p^{-1} \hat{P}_u^{-1}$$

(26)

$$\beta_u^* = \|Q_u (I - P_u \hat{P}_u^{-1}) S_p\|_\infty < 1$$

(27)

and with complete learning (i.e., $Q_u = I$), the inner plant behaves like the nominal model as the model following error $e_{p,\infty} \rightarrow 0$.

**Convergence of Tracking Error** Using (17) and (25), the tracking error $e_k$ can be derived as

$$e_k = P_u T_u^{-1} \left[ e_{p,k} + \hat{P}_{mu} C (P_{u}^{-1} P_{mu} \hat{P}_u^{-1} - I) q_{d,k} + (\hat{P}_{m_u}^{-1} + C) \hat{P}_{u,q,k} + C \hat{P}_{u} (P_{m_u} - P_{d} P_{m_d} P_{u}^{-1}) d_k \right]$$

(28)

By assumption, $q_{d,k}$, $r_{q,k}$, and $d_k$ do not vary from one iteration to another. Thus the tracking error $e_k$ will also converge monotonically with the rate of $\beta_u \leq \|P_u T_u^{-1}\|_\infty \beta_u$, if the model following error $e_{p,k}$ converges and $P_u T_u^{-1}$ is BIBO stable.

Note that for torque ILC (M), $P_{m_u}^{-1} \hat{P}_{m_u} \hat{P}_u^{-1} - I = 0$ and $P_{m_d} - P_d P_{m_d} P_{u}^{-1} = 0$, which further reduces (28) to

$$e_k = P_u T_u^{-1} \left[ e_{p,k} - (\hat{P}_{m_u}^{-1} + C) \hat{P}_{u,q,k} \right]$$

(29)

Thus if $r_{q,k} = 0$ (i.e., the reference ILC is not activated), $e_k \rightarrow 0$ as $e_{p,k}$ vanishes. For torque ILC (L), this is not true due to the mismatched behavior. The remaining tracking error $e_{\infty}$ is

$$e_{\infty} = P_u T_u^{-1} \left[ \hat{P}_{mu} C (P_{u}^{-1} P_{mu} \hat{P}_u^{-1} - I) q_{d,\infty} + (\hat{P}_{m_u}^{-1} + C) \hat{P}_{u,q,\infty} + C \hat{P}_{u} (P_{m_u} - P_{d} P_{m_d} P_{u}^{-1}) d_{\infty} \right]$$

$$\triangleq -P_{cu,u} r_{q,\infty} + \bar{r}_{u,\infty}$$

(30)

which can be further eliminated through the reference ILC using $L_u^* = \hat{P}_{u,u}^{-1} \hat{P}_u$, and this matches with (19).

**Hybrid Scheme With Two-Stage ILC**

In general, for the closed loop system with a satisfactory feedback controller, the sensitivity function $S_p$ will behave as a high-pass filter to mitigate the low frequency error. Therefore,
in the convergence condition (27), the low frequency model uncertainty is greatly suppressed by $S_p$. This allows $Q_u$ to have higher bandwidth without worrying about the low frequency uncertainty. Then with the effects of the torque ILC, the inner plant will behave like the nominal model (i.e., $q_k \rightarrow \hat{P}_u k$) up to the bandwidth of $Q_u$. Within this frequency range, the convergence condition of the reference ILC (20) will be simplified to

$$\beta_r \approx \|Q_r (I - S_p \hat{P}^{-1})\|_\infty < 1$$

which allows to push $Q_r$ to a higher bandwidth for better tracking performance.

Note that the repetitive assumption is used in the derivation of the aforementioned two ILC schemes. When these two ILC schemes are activated simultaneously, the repetitive assumption will be no longer valid (i.e., $r_q k$ and $\tau_{nl,k}$ are not repetitive from one iteration to another). Therefore, an ad hoc hybrid scheme is designed to reduce the adverse interference of the two ILC stages. Specifically, an iteration-varying gain is applied to each ILC stage as follows

$$\tau_{nl,k+1} = Q_u (\tau_{nl,k} + \gamma_{nl,k} L \omega_{p,k})$$
$$r_{q,k+1} = Q_r (r_{q,k} + \gamma_{q,k} L \omega_{e,k})$$

where the two gains $\gamma_{nl,k}$ and $\gamma_{q,k}$ can be tuned by trial and error, e.g., $\gamma_{nl,k} = \min(4 \sum \|r_{p,k}(j)|^2 \|, 1)$ and $\gamma_{q,k} = 1 - \frac{1}{2} \gamma_{nl,k}$. The basic idea behind is that the torque ILC needs to take more effects whenever the model following error becomes larger in the previous iteration (i.e., $\sum \|r_{p,k}(j)|^2 \|$ increases). In order for the torque ILC to perform better, the effects of the reference ILC is accordingly attenuated with a decreased $\gamma_{nl,k}$. In contrast, once the model following error is sufficiently small (i.e., the inner plant behaves as the nominal model), the torque ILC becomes not necessary and the reference ILC can be fully activated.

As shown in (29), for the application of tracking $q_m$, the reference ILC is not necessary and the torque ILC (M) with $\hat{P}_u = \hat{P}_mu$ will be sufficient. In order to track $q_t$, however, the aforementioned hybrid two-stage ILC scheme with $\hat{P}_u = \hat{P}_tu$ will be necessary. And it is understood that the nominal models used in two ILC stages should match with each other due to the mismatched dynamics. In the experimental study, the algorithm validation will focus on the case of tracking $q_t$ to test the effectiveness of the hybrid two-stage ILC scheme.

**EXPERIMENTAL STUDY**

**Experimental Setup & Dynamic Model**

The proposed method is validated on a single-joint indirect drive robot shown in Fig. 2. This experimental setup consists of:

- 1) a servo motor with 20,000 counts/revolution encoder, 2) a harmonic drive with a 80:1 gear ratio, 3) a load side 144,000 counts/revolution encoder, and 4) a payload. The anti-resonant and resonant frequencies of the setup are approximately 11Hz and 19Hz. It is assumed that the load side encoder measurement is only available for iteration based offline use rather than for real-time feedback use. Finally, the algorithms are implemented using a 1kHz sampling rate in a LabVIEW real-time target installed with LabVIEW Real-Time and FPGA modules.

Figure 2 also illustrates the schematic of the single-joint indirect drive mechanism. The subscripts $m$ and $l$ denote the motor side and the load side quantities, respectively. $\theta$ represents the angular position and $J$ is the moment of inertia. $u$ is the motor torque input. $d_m$ and $d_l$ represent the viscous damping coefficients at the motor side and the load side, respectively. $k_j$ and $d_j$ are the stiffness and the damping coefficients of the reducer. The gear ratio of the reducer is denoted by $N$. $f_{mc}$ and $f_{fe}$ represent the nonlinear Coulomb friction coefficients at the motor side and the load side, respectively. $\hat{\theta} = 2 \theta (2 \theta_{nt} + \phi)$, where $A$ is the amplitude of the transmission error, $\phi$ is the phase, and the frequency is 2 times the motor side velocity [12].

The dynamic model for this setup can be formulated as

$$J_m \ddot{\theta}_m + d_m \dot{\theta}_m = u + d_1 \left[ k_j \left( \frac{\theta_m}{N} - \theta_l \right) + d_j \left( \frac{\theta_m}{N} - \theta_l \right) \right]$$
$$J_l \ddot{\theta}_l + d_l \dot{\theta}_l = k_j \left( \frac{\theta_m}{N} - \theta_l \right) + d_j \left( \frac{\theta_m}{N} - \theta_l \right) + d_2$$
where

\[
\begin{align*}
    d_1 &= d_{fm} - f_m \sec(\theta_m) + \frac{1}{N} \left( k_i \dot{\theta} + d_j \dot{\theta} \right) \\
    d_2 &= d_{fl} - f_l \sec(\theta_l) - \left( k_i \dot{\theta} + d_j \dot{\theta} \right)
\end{align*}
\]

Therefore, the above indirect drive model can be considered as a mismatched system described in (3)-(4) with the disturbance \( d = [d_1, d_2]^{T} \). The two outputs \( q_m \) and \( q_l \) are the motor side position \( \theta_m \) and the load side position \( \theta_l \), respectively. Note that \( d \) is repetitive if \( q_m \) and \( q_l \) are repetitive. The continuous time transfer functions from the inputs to the outputs become

\[
\begin{align*}
    P_{mu}(s) &= \frac{J_l s^2 + (d_j + d_l)s + k_j}{J_m J_l s^4 + J_d s^3 + J_k s^2 + k_j (d_m + d_l/N^2)s} \\
    P_{tu}(s) &= \frac{d_j s + k_j}{N [J_m J_l s^4 + J_d s^3 + J_k s^2 + k_j (d_m + d_l/N^2)s]} \\
    P_{mls}(s) &= \frac{d_j s + k_j}{N [J_m J_l s^4 + J_d s^3 + J_k s^2 + k_j (d_m + d_l/N^2)s]} \\
    P_{lds}(s) &= \frac{J_m s^2 + (d_j/N^2 + d_m)s + k_j/N^2}{J_m J_l s^4 + J_d s^3 + J_k s^2 + k_j (d_m + d_l/N^2)s} \\
    P_{md}(s) &= [P_{mu}(s) \quad P_{mls}(s)] , \quad P_{ld}(s) = [P_{tu}(s) \quad P_{lds}(s)]
\end{align*}
\]

System Disturbance Characteristics

In this setup, the identified Coulomb friction combined at the motor side (i.e., \( f_m + \frac{d_m}{N^2} \)) is about 0.1004Nm. A fictitious torque is added in the motor torque command to simulate the external disturbance \( d_{fm} \) at the motor side. In the following experiments, \( d_{fm} \) is set as a 1Hz sinusoid starting from 3 sec with an amplitude of 0.2Nm, i.e., \( d_{fm} = 0.2 \sin(2\pi(t - 3)) \) Nm. The repetitive external disturbance \( d_{fl} \) at the load side is generated using the setup shown in Fig. 3. It is designed to have the extension springs apply a maximum disturbance of approximately 20 Nm at the load side when the payload hits the ball and continues rotating for about 14 degrees.

The motor side feedback controller \( C \) designed for this setup has a resonant frequency at about 11Hz for the velocity loop. This corresponds to about 1 rad/sec at the load side. Therefore, in order to amplify the transmission error effects, the load side desired trajectory is designed to have a speed of 0.5 rad/sec for most time so that the resulting transmission error frequency will coincide with the resonant frequency of the velocity loop. The resulting trajectory is shown in Fig. 4, which is designed as a fourth-order time optimal trajectory suggested in [13].

The effects of these different kinds of disturbances on the load side tracking performance with the basic controller structure (i.e., \( C, F_1, \) and \( F_2 \) in Fig. 1) are illustrated in Fig. 5.

Algorithm Setup

The zero-phase acausal low-pass filters \( Q_r \) and \( Q_u \) are obtained as \( Q_r(z) = Q_u(z) = Q_1(z^{-1})Q_1(z) \), where \( Q_1(z) \) is a low-pass filter with a cut-off frequency of 30Hz, which is beyond the system elastic anti-resonant and resonant frequencies. With this selection of \( Q_r(z) \) and \( Q_u(z) \), the frequency responses of \( \beta_r \) in (20) and \( \beta_u \) in (27) with \( \pm 50\% \) parametric uncertainties are plotted in Fig. 6 to verify the monotonic stability condition.

Figure 6 shows that, using either motor side model or load side model, the magnitudes of \( \beta_r \) and \( \beta_u \) are always below 0dB indicating \( \beta_r < 1 \) and \( \beta_u < 1 \). Therefore, the monotonic stabilities (20) and (27) are ensured separately for both ILC schemes.
Performances in the following experiments will be compared in rate the ILC convergence performance. To see this, the tracking attenuate the model uncertainty but instead may deteriorate well. As shown in Fig. 7, the load side position tracking errors for these three settings are all significantly reduced to almost the level of load side encoder resolution.

Experimental Results

Each controller setting is implemented to track the load side desired trajectory in Fig. 4 for 10 iterations. First, the nominal model with accurately identified system dynamic parameters is used in the controller design. With an accurate nominal model, it is expected that the three controller settings will perform equally well. As shown in Fig. 7, the load side position tracking errors for these three settings are all significantly reduced to almost the level of load side encoder resolution.

Next the three controller settings are compared using the nominal model with 15% uncertainty in the dynamic parameters. Normally, with larger model uncertainties, the cut off frequencies of Q filters need to be reduced to ensure the convergence of the learning process. Here, the Q filters are kept the same as the previous case in order to verify the benefits of the proposed scheme. Fig. 8(a) shows that the torque ILC performs very well once it is activated. No matter which nominal model is chosen to follow, the model following errors are greatly reduced with fast convergence rate. As to the load side tracking performance, however, more differences are expected. It is shown in Fig. 8(b) that, due to the model uncertainty, the setting RefILC(L) does not perform as well as before. The setting RefILC(L)+TrqILC(M) actually deteriorates the performance since TrqILC(M) is intended to make the inner plant match with the motor side nominal model while the load side behavior may actually deviate further from its nominal behavior. In contrast, the setting RefILC(L)+TrqILC(L) performs the best since it intends to make the inner plant behave as the load side nominal model and thus greatly releases the uncertainty burden on the reference ILC.

Now consider the two-stage approach proposed in [6]. It can be shown that, the approach in [6] with plant inversion learning filters can be reformulated similarly as the reference ILC (L) with \( \hat{P}_u = \hat{P}_{u_m} \) plus the torque ILC (M) with \( \hat{P}_u = \hat{P}_{u_m} \) in this paper. This means that the two-stage ILC scheme is performed with mismatched nominal models. As expected, this will not help to attenuate the model uncertainty but instead may even deteriorate the ILC convergence performance. To see this, the tracking performances in the following experiments will be compared in three controller settings, i.e., reference ILC with \( \hat{P}_u = \hat{P}_{u_m} \) only (RefILC(L)), reference ILC with \( \hat{P}_u = \hat{P}_{u_M} \) plus torque ILC with

\[
\beta_r = \| Q_r (1 - \hat{P}_u^{-1} P_y S_p^{-1}) \|_{\infty} < 1
\]

\[
\beta_u = \| Q_u (1 - \hat{P}_u^{-1} P_y S_p^{-1}) \|_{\infty} < 1
\]

\[\hat{P}_u = \hat{P}_{u_m} \text{(RefILC(L) + TrqILC(M))}, \text{and reference ILC with } \hat{P}_u = \hat{P}_{u_M} \text{ plus torque ILC with } \hat{P}_u = \hat{P}_{u_m} \text{(RefILC(L) + TrqILC(L))}.\]
CONCLUSIONS

In this paper, a model based two-stage ILC scheme was developed for a class of MIMO mismatched linear systems. To improve the performance of the ILC stage aimed for tracking error reduction, another ILC utilizing the idea of model following was designed to drive the inner plant to behave like the nominal model. The convergence property was investigated with proper $Q$ filter and learning filter design. In order to make the two ILC stages work properly together, an ad hoc hybrid scheme was proposed to make transitions between the two ILC stages. A single-joint indirect drive system with several inherent and designed external disturbances was utilized to experimentally validate the proposed ILC scheme. It was shown that the proposed hybrid two-stage ILC scheme using the load side nominal model outperformed the other two benchmark controller settings in the load side tracking application. The application extension of this work to a 6-joint robot manipulator has also been conducted and the results will be reported in [14].

REFERENCES


